

## **METHOD OF GENERATING SINUSOIDAL SIGNAL**

### **FIELD OF THE INVENTION**

**[0001]** The invention relates to digitally generating a sinusoidal signal, in particular to generating a sinusoidal signal by using real-number arithmetic and processing with multiple sampling frequencies.

### **BACKGROUND OF THE INVENTION**

**[0002]** Digital sine-wave generators are commonly used for generating an oscillating sinusoidal signal in a variety of electronic applications. In mobile stations, for instance, digital sine-wave generators are typically used for generating simple ringing tones or DTMF (Dual-Tone-Multi-Frequency) tones among other things.

**[0003]** In accordance with the prior art, a sinusoidal signal can be generated digitally, for instance, by using a signal waveform stored in a lookup table. The method is commonly used in a plurality of electronic devices, such as mobile stations, in which sine-wave samples are stored in the tables. It is also possible to store in the tables other waveforms, such as tone samples. The method typically employs a counter to generate a table address, at which a desired tone sample can be found when needed. However, if there are a large number of stored samples, the tables may become very large, which requires that the memory capacity of the electronic device be large.

**[0004]** The size of the table can yet be minimized by storing in the table a sample, whose length is only one quarter of the wave length of the sinusoidal signal. By combining these quarters of the sinusoidal signal it is possible to generate various DTMF tones, for instance. In addition, it is possible to store coarse samples and to interpolate finer samples. Nevertheless, the problem remains that the memory capacity of the electronic device is used excessively. Currently, one major criterion for designing electronic devices, in particular, mobile stations, is to minimize the use of memory capacity.

**[0005]** A sinusoidal signal can also be generated digitally by using a prior art digital sine-wave oscillator. Typical sine-wave oscillators require at least one coefficient, i.e. for each sample there is at least one multiplication operation, which determines a frequency to be generated. Because the digital sine-wave oscillators typically have a predetermined, arithmetic total term length, the coefficient should be generally quantized, prior to generating the sinusoidal signal, to have a length that matches with space allocated to it.

However, the quantization of the coefficient distorts the sinusoidal signal to be generated, which affects both the frequency and amplitude of the signal. Hence, it is generally difficult to generate precisely a frequency in the vicinity of a low frequency or a half sampling rate.

**[0006]** The effect of the quantization can be reduced by increasing filter bit widths so as to achieve sufficient performance. However, a problem arises that the method considerably increases computational complexity. In addition, bit width modifications and scalings generally restrict parametrization of a sinusoidal signal.

**[0007]** The publication "A Simple Recursive Digital Sinusoidal Oscillator with Uniform Frequency Spacing", The 2001 IEEE International Symposium on Circuits and Systems, Part:2, 2001, by M.M.Al-Ibrahim, pp. 689 to 692, presents a digital sine-wave oscillator used in telecommunications applications, which oscillator is arranged to generate sinusoidal signals, advantageously low-frequency sinusoidal signals with even frequency spacing. A problem with the sine-wave oscillator of the publication is, however, that it employs complicated complex number arithmetic for generating a sinusoidal signal. If an algorithm implementing the functionality according to the publication is executed with a digital signal processor DSP, for instance, it will be necessary to perform several command cycles for each output sample.

## BRIEF DESCRIPTION OF THE INVENTION

**[0008]** It is an object of the invention to provide a procedure that allows reduction of the above-described problems. This is achieved with methods, devices and a software product, which are characterized by what is disclosed in the independent claims. The preferred embodiments of the invention are disclosed in the dependent claims.

**[0009]** The invention is based on determining a desired frequency  $f$  and a sampling rate  $f_s$  for a sinusoidal signal to be generated. If the desired frequency  $f$  is higher than an upper limit frequency, advantageously higher than  $0.375f_s$ , a coefficient  $c$  is determined as a function of a multiple  $N$  of the sampling rate  $f_s$ , for instance such that the value of the coefficient  $c$  is given by the equation (1):

$$c = 2 \cos \left( \frac{2\pi f}{Nf_s} \right) \quad (1)$$

**[0010]** The  $n^{\text{th}}$  sample of the first output sample sequence is determined as a linear combination of the coefficient  $c$  and two previous output samples. The output sample sequence is decimated by a multiple  $N$  of the sampling rate  $f_s$  so as to generate a sinusoidal signal of the desired frequency  $f$  at a desired sampling rate  $f_s$ .

**[0011]** On the other hand, the invention is based on determining a desired frequency  $f$  and a sampling rate  $f_s$  for a sinusoidal signal to be generated. If the desired frequency  $f$  is lower than a lower limit frequency, advantageously lower than  $0.125f_s$ , the coefficient  $c$  is determined as a function of a multiple  $N$  of the sampling rate  $f_s$ , for instance such that the value of the coefficient  $c$  is given by the equation (2):

$$c = 2 \cos \left( \frac{1}{N} \left( \pi - \frac{f}{f_s} \right) \right) \quad (2)$$

The  $n^{\text{th}}$  sample of the first output sample sequence is determined as a linear combination of the coefficient  $c$  and two previous output samples. The first output sample sequence is multiplied by a fixed-frequency sine wave so as to generate a second output sample sequence. The second output sample sequence is decimated by a multiple  $N$  of the sampling rate  $f_s$  so as to generate a sinusoidal signal of a desired frequency  $f$  at a desired sampling rate  $f_s$ .

**[0012]** According to a preferred embodiment the coefficient  $c$  is determined by means of a discrete frequency index,  $\text{findex}$ , such that the value of the frequency index,  $\text{findex}$ , substantially corresponds to the desired frequency  $f$ .

**[0013]** Considerable advantages are achieved with the arrangement of the invention. One advantage is that determination of an output sample is relatively simple, because the procedure employs real-number arithmetic instead of complicated complex-number arithmetic. A further advantage is that the procedure of the invention allows relatively precise generation of a sinusoidal signal of a desired frequency in the vicinity of a low frequency and a half sampling rate, because the coefficient  $c$  that determines the generated frequency can represent relatively precisely the frequency  $f$  in this frequency range. An advantage is also that the procedure does not require lookup tables, which consume memory capacity and which can be very large in size.

## BRIEF DESCRIPTION OF THE DRAWINGS AND APPENDICES

**[0014]** In the following the invention will be described in greater detail in connection with embodiments, with reference to the attached drawings, in which

Figure 1 illustrates a prior art digital sine-wave oscillator;

Figure 2 illustrates a digital sine-wave oscillator for high frequencies in accordance with one preferred embodiment of the invention;

Figure 3 illustrates a digital sine-wave oscillator for low frequencies in accordance with one preferred embodiment of the invention; and

Figure 4 shows the relation between frequencies generated by various methods as a function of frequency index.

## DETAILED DESCRIPTION OF THE INVENTION

**[0015]** Digital sine-wave generators, such as digital sine-wave oscillators, are commonly used for generating an oscillating sinusoidal signal in a variety of electronic applications. In mobile stations, for instance, the digital sine-wave generators are typically used for generating simple ringing tones or DTMF tones among other things.

**[0016]** The operation of the sine-wave oscillators is generally based on feedback coupling, because an output signal is applied as feedback to an input of a circuit. When coupled, supply voltage produces a voltage pulse in the output signal. When the pulse is fed back to the input, it is typically amplified and re-coupled back to the input. Thus the coupling starts oscillating. The sine-wave oscillators include, for instance, RC oscillators, such as Wien bridge oscillators, and RC chain oscillators, LC oscillators and crystal oscillators. The invention and the embodiments thereof are not restricted to the presented oscillators, however, but the invention and its embodiments can also be applied to other sine-wave generators.

**[0017]** In connection with the present invention the sampling rate  $f_s$  refers to how often the signal is sampled, i.e. how often the analogue signal is digitized. The quality of the digitized signal generally improves, as the sampling rate  $f_s$  increases.

**[0018]** In the present document the preferred embodiments of the invention are described using a double sampling rate  $2f_s$ . However, the invention and its preferred embodiments are not restricted to the use of a double sampling rate  $2f_s$ , but the procedure according to the invention can also be im-

plemented by using any multiple  $N$  of the sampling rate  $f_s$ . In that case the samples are decimated by the coefficient of the multiple  $N$  so as to generate a sinusoidal signal of a desired frequency  $f$ .

**[0019]** Figure 1 is a block diagram of a typical digital sine-wave generator, in which the  $n^{\text{th}}$  sample  $y(n)$  of the output sample sequence is a linear combination of two previous output samples,  $y(n-1)$  and  $y(n-2)$ . This can be described by the following recursive difference equation (3):

$$y(n) = c \cdot y(n-1) - y(n-2) \quad (3)$$

where

$n \geq 0$

$y(n)$  = the  $n^{\text{th}}$  sample of the output sample sequence

$c$  = coefficient

**[0020]** The coefficient  $c$  can be determined as a function of a desired frequency  $f$  and the sampling rate  $f_s$ , for instance by the following equation (4):

$$c = 2 \cos\left(\frac{2\pi f}{f_s}\right) \quad (4)$$

**[0021]** The coefficient  $c$  is typically quantized in advance, because the length of the coefficient  $c$  must not exceed the length allocated to it in the digital sine-wave oscillator, the total term length of the oscillator arithmetic being predetermined. When the coefficient  $c$  is determined by the equation (4), there is problem that, due to the cosine function, the quantized coefficient  $c$  does not generally represent with high accuracy a frequency  $f$  that is very low as compared with the sampling rate  $f_s$  or which is close to a half sampling rate  $f_s/2$ . However, the problem can be solved by frequency conversion, for instance, as a result of which it is possible to obtain a sinusoidal signal relatively precisely at the desired frequency  $f$ . The frequency conversion increases the complexity of the procedure, however.

**[0022]** On the other hand, it is also possible to generate sinusoidal signals, for instance, with complicated, arithmetic sine-wave generators by combining the output samples of e.g. two sine-wave generators, of which the

first generator generates an adjustable frequency and the second generator generates a fixed frequency, whereby the generator generating the adjustable frequency can operate in a frequency range, where the quantized coefficient  $c$  may represent the frequency with higher accuracy.

**[0023]** The procedure of the invention is based, in part, on the latter procedure. However, the procedure of the invention is considerably simpler and therefore more readily applicable to various uses. The procedure of the invention utilizes real-number arithmetic and processing of a plurality of sampling rates in order to generate a sinusoidal signal having desired characteristics.

**[0024]** Figure 2 is a block diagram of a digital sine-wave oscillator, by which it is possible to implement a method in accordance with a preferred embodiment, which is described next.

**[0025]** The method predetermines the desired frequency  $f$  and the sampling rate  $f_s$  of the sinusoidal signal to be generated. In the present document a lower limit frequency refers to a frequency that is substantially  $0.2f_s$  and an upper limit frequency refers to a frequency that is substantially  $0.3f_s$ . If the desired frequency  $f$  is higher than the upper limit frequency, advantageously higher than  $0.375f_s$ , a coefficient  $c$  is determined in the following manner. Because the sinusoidal signal is sampled at a double sampling rate  $2f_s$ , in order that the relation between the generated frequency  $f$  and the double sampling rate  $2f_s$ , i.e. the so-called upper sampling rate, would be within a range, where the frequencies can be represented relatively precisely with quantized arithmetic, the coefficient  $c$  is determined as a function of the generated frequency  $f$  and the sampling rate  $f_s$  according to the equation (4) such that twice the sampling rate  $2f_s$  is placed as the sampling rate  $f_s$ , whereby the equation (4) will have the form (1) of

$$c = 2 \cos \left( \frac{2\pi f}{Nf_s} \right) \quad (1)$$

where  $N$  represents a multiple of the sampling rate  $f_s$ . In this preferred embodiment  $N = 2$ . The  $n^{\text{th}}$  sample of the first output sample sequence  $y_1(n)$  is generated as a linear combination of the coefficient  $c$  and two previous output samples  $y_1(n-1)$  and  $y_1(n-2)$ . As described above, the sinusoidal signal is sampled at a higher sampling rate  $2f_s$ . The first output sample sequence  $y_1$  that was sampled at the higher sampling rate  $2f_s$  is decimated  $D$  by two, i.e. only

every second sample will be taken for generating the sinusoidal signal of desired frequency at the desired sampling rate  $f_s$ .

**[0026]** Figure 3 shows a block diagram of a digital sine-wave oscillator that implements a method of an advantageous embodiment to be described next.

**[0027]** The method predetermines the desired frequency  $f$  and the sampling rate  $f_s$  of the sinusoidal signal to be generated. If the desired frequency  $f$  is lower than the lower limit frequency, advantageously lower than  $0.125f_s$ , the coefficient  $c$  is determined in the following manner. Because the method first generates the sinusoidal signal to frequency  $0.5f_s - f$  and the sinusoidal signal is sampled at a double sampling rate  $2f_s$ , in order that the relation between the generated frequency  $f$  and the double sampling rate  $2f_s$ , i.e. the so-called upper sampling rate, would be within a range, where the frequencies can be represented relatively precisely with quantized arithmetic, the coefficient  $c$  is determined as a function of the generated frequency  $f$  and the sampling rate  $f_s$  according to the equation (4) such that the frequency  $0.5f_s - f$  is placed as the frequency  $f$ , whereby the equation (4) will have the form (2) of

$$c = 2 \cos \left( \frac{1}{N} \left( \pi - \frac{f}{f_s} \right) \right) \quad (2)$$

**[0028]** The  $n^{\text{th}}$  sample  $y_1(n)$  of the first output sample sequence is generated to frequency  $0.5f_s - f$  as a linear combination of the coefficient  $c$  and two previous output samples  $y_1(n-1)$  and  $y_1(n-2)$ . The frequency  $0.5f_s - f$  is close to a quarter of the upper sampling rate  $2f_s$ , when the desired frequency  $f$  is low. As described above, the sinusoidal signal is sampled at the double sampling rate  $2f_s$ , as a result of which the first generated output sample sequence  $y_1$  is multiplied by a fixed-frequency sine wave, such as the sequence 1, 0, -1, 0, 1, 0, -1, ..., which substantially corresponds to multiplication by another sinusoidal signal whose frequency is one quarter of the upper sampling rate, i.e.  $f_s/2$ . Due to the real-number arithmetic, multiplication of the first output sample sequence  $y_1$  by this fixed-frequency sine wave produces two sine-wave components, i.e. the second output sample sequence  $y_2$ . The frequency of the first component is the desired frequency  $f$  and the frequency of the second component is the difference between the sampling rate  $f_s$  and the desired frequency  $f$ , i.e.  $f_s - f$ . The second output sample sequence  $y_2$  will be deci-

mated D by two, whereby the second sine-wave component aliases to the desired frequency  $f$ , whereby one sinusoidal signal of the desired frequency  $f$  at the desired sampling rate  $f_s$  will be obtained.

**[0029]** According to a preferred embodiment the coefficient  $c$  is determined by using a discrete frequency index,  $findex$ , whereby it is possible to avoid the calculation of the coefficient  $c$  in the sine-wave oscillator by the equation (4). The use of the frequency index,  $findex$ , is illustrated in the following example.

**[0030]** When the frequency index,  $findex$ , is an integer within the range of 0 to 65535, the prior art generation method of a sinusoidal signal, such as the sine-wave generator of Figure 1, allows the generation of the following frequencies given by the equation (5) derived from the equation (5):

$$\frac{f}{f_s} = \frac{1}{2\pi} \arccos\left(\frac{32768 - findex}{32768}\right) \quad (5)$$

**[0031]** The parenthetical expression corresponds here to half the coefficient  $c$ , i.e.  $c/2$ .

**[0032]** The prior art generation method of the sinusoidal signal is advantageously used for generating sinusoidal signals, whose desired frequency  $f$  is within the frequency range of  $0.125f_s$  to  $0.375f_s$ , whereby  $16384 \leq findex \leq 49151$ . Thus the relation between the desired frequency  $f$  and the sampling rate  $f_s$  is within a range, where the coefficient  $c$  may represent relatively precisely the desired frequency  $f$ .

**[0033]** When the desired frequency  $f$  is higher than the upper limit frequency, advantageously higher than  $0.375f_s$ , whereby  $49151 \leq findex \leq 65535$ , the relation between the frequency  $f$  and the sampling frequency  $f_s$  is obtained by deriving from the equation (4):

$$\frac{f}{f_s} = \frac{1}{\pi} \arccos\left(\frac{65535 - findex}{32768}\right) \quad (6)$$

**[0034]** However, if the desired frequency  $f$  is lower than the lower limit frequency, advantageously lower than  $0.125f_s$ , whereby  $0 \leq findex \leq 16384$ , the relation between the desired frequency  $f$  and the sampling rate  $f_s$  is obtained by deriving from the equation (2):



$$\frac{f}{f_s} = \frac{1}{2} - \frac{1}{\pi} \arccos\left(\frac{findex}{32768}\right) \quad (7)$$

**[0035]** According to a preferred embodiment the sinusoidal signal generation functionality can also be implemented by an algorithm, which is typically coded into a software code with a programming language, such as C++. The software code can be executed, for instance, with a digital signal processor DSP or a micro controller unit MCU. The same arithmetic functionality can also be executed with a fixed logic, such as an ASIC circuit, for instance.

**[0036]** The diagram of Figure 4 illustrates the above-described embodiments. The curve 1 represents the relation of the frequencies generated by the prior art method and the curve 2 that of the frequencies generated by the above-described preferred embodiments as a function of the frequency index, findex. The curve 3 represents the relation of the frequencies as a function of the frequency index, findex, in an ideal case. When the results obtained by various methods are compared with the ideal curve, it can be noted that the methods of the preferred embodiments provide better results than the prior art sinusoidal signal generation method, if the generated frequency is lower than the lower limit frequency or higher than the upper limit frequency. The prior art generation method may, however, provide results that correspond to those obtained by the preferred embodiments, if the generated frequency is higher than or equal to the lower limit frequency, or lower than or equal to the upper limit frequency.

**[0037]** The above-described generation methods of the sinusoidal signal can be implemented by the sine-wave generators according to preferred embodiments of the invention.

**[0038]** According to one preferred embodiment, a sine-wave generator comprises means for determining the  $n^{\text{th}}$  sample  $y_1(n)$  of the first output sample sequence as a linear combination of said coefficient  $c$  and two previous output samples  $y_1(n-1)$  and  $y_1(n-2)$ . The sine-wave generator also comprises means for determining the relation between the desired frequency  $f$  and the sampling rate  $f_s$ , means for determining the coefficient  $c$  as a function of a multiple  $N$  of the sampling rate  $f_s$ , for instance, such that its value corresponds to the equation (1) presented above, and means for decimating the first output

sample sequence  $y_1(n)$  by the multiple  $N$  of the sampling rate  $f_s$  for generating the sinusoidal signal of the desired frequency  $f$  at the desired sampling rate  $f_s$  in response to the desired frequency  $f$  being higher than the upper limit frequency.

**[0039]** According to one preferred embodiment, a sine-wave generator comprises means for determining the  $n^{\text{th}}$  sample  $y_1(n)$  of the first output sample sequence as a linear combination of said coefficient  $c$  and two previous output samples  $y_1(n-1)$  and  $y_1(n-2)$ . The sine-wave generator also comprises means for determining the relation between the desired frequency  $f$  and the sampling rate  $f_s$ , means for determining the coefficient  $c$  as a function of a multiple  $N$  of the sampling rate  $f_s$ , for instance, such that its value corresponds to the equation (2) presented above, means for multiplying the first output sample sequence  $y_1$  by a fixed-frequency sine wave, such as the sequence 1, 0, -1, 0, 1, 0, -1,..., for generating a second output sample sequence  $y_2$ , and means for decimating the second output sample sequence  $y_2$  by the multiple of the sampling rate  $f_s$  so as to generate the sinusoidal signal of the desired frequency  $f$  at the desired sampling rate  $f_s$  in response to the desired frequency  $f$  being lower than the lower limit frequency.

**[0040]** According to one preferred embodiment, a sine-wave oscillator comprises means for determining the  $n^{\text{th}}$  sample  $y_1(n)$  of the first output sample sequence as a linear combination of the coefficient  $c$  and two previous output samples  $y_1(n-1)$  and  $y_1(n-2)$ . The sine-wave oscillator also comprises means for determining the relation between the desired frequency  $f$  and the sampling rate  $f_s$ , means for determining the coefficient  $c$  as a function of a multiple  $N$  of the sampling rate  $f_s$ , for instance, such that its value corresponds to the equation (1) presented above, and means for decimating the first output sample sequence  $y_1$  by the multiple  $N$  of the sampling rate  $f_s$  for generating the sinusoidal signal of the desired frequency  $f$  at the desired sampling rate  $f_s$  in response to the desired frequency  $f$  being higher than the higher limit frequency. The sine-wave oscillator also comprises means for determining the coefficient  $c$  as a function of the sampling rate  $f_s$ , for instance, such that its value corresponds to the equation (2) presented above, means for multiplying the first output sample sequence  $y_1$  by a fixed-frequency sine wave, such as the sequence 1, 0, -1, 0, 1, 0, -1,..., so as to generate a second output sample sequence  $y_2$ , and means for decimating the second output sample sequence  $y_2$  by the multiple  $N$  of the sampling rate  $f_s$  so as to generate the sinusoidal signal

of the desired frequency  $f$  at the desired sampling rate  $f_s$  in response to the desired frequency  $f$  being lower than the lower limit frequency. The sine-wave oscillator also comprises means for determining the coefficient  $c$  as a function of the sampling rate  $f_s$ , for instance, such that its value corresponds to the equation (4) presented above, so as to generate the sinusoidal signal of the desired frequency  $f$  at the desired sampling rate  $f_s$  in response to the desired frequency  $f$  being higher than or equal to the lower limit frequency or lower than or equal to the upper limit frequency.

**[0041]** It is also possible to provide the generation functionality of the sinusoidal signal by a software product that can be arranged in an electronic device.

**[0042]** According to one preferred embodiment, the software product comprises a program code for determining the  $n^{\text{th}}$  sample  $y_1(n)$  of the first output sample sequence as a linear combination of the coefficient  $c$  and two previous output samples  $y_1(n-1)$  and  $y_1(n-2)$ , and a program code for determining the desired frequency  $f$  and the sampling rate  $f_s$ . The software product also comprises a first sub-process in response to the desired frequency  $f$  being higher than the upper limit frequency. The first sub-process comprises a program code for determining the coefficient  $c$  as a function of a multiple of the sampling rate  $f_s$ , for instance, such that its value corresponds to the equation (1) presented above, and a program code for decimating the first output sample sequence  $y_1$  by the multiple  $N$  of the sampling rate  $f_s$  at the desired sampling rate  $f_s$ . The software product also comprises a second sub-process in response to the desired frequency  $f$  being lower than the lower limit frequency. The second sub-process comprises a program code for determining the coefficient  $c$  as a function of the multiple  $N$  of the sampling rate  $f_s$ , for instance, such that its value corresponds to the equation (2) presented above, a program code for multiplying the first output sample sequence  $y_1$  by a fixed-frequency sine wave, such as the sequence 1, 0, -1, 0, 1, 0, -1, ..., so as to generate the second output sample sequence  $y_2$  and a program code for decimating the second output sample sequence  $y_2$  by the multiple  $N$  of the sampling rate  $f_s$  so as to generate the sinusoidal signal of the desired frequency  $f$  at the desired sampling rate  $f_s$ . The software product also comprises a third sub-process in response to the desired frequency  $f$  being higher than or equal to the lower limit frequency or lower than or equal to the upper limit frequency. The third sub-process comprises a program code for determining the coefficient  $c$  as a

function of the multiple  $N$  of the sampling rate  $f_s$ , for instance, such that its value corresponds to the equation (4) presented above so as to generate the sinusoidal signal of the desired frequency  $f$  at the desired sampling rate  $f_s$ .

**[0043]** It is apparent to a person skilled in the art that as technology progresses, the basic idea of the invention can be implemented in a variety of ways. Thus the invention and its preferred embodiments are not restricted to the above-described examples and components, but they may vary within the scope of the claims.